National Open School of Trinidad and Tobago

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Eight:
Functions and Relations

Lesson 17

Inverting Functions

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2.
(Some helpful exercises and page numbers are given throughout the lesson, e.g. (Ex 14b page 806)

INTRODUCTION

In this lesson we will be looking at finding the inverse of functions. This is an important skill that can be used to identify elements in the domain that members of the range are mapped onto.

OBJECTIVES

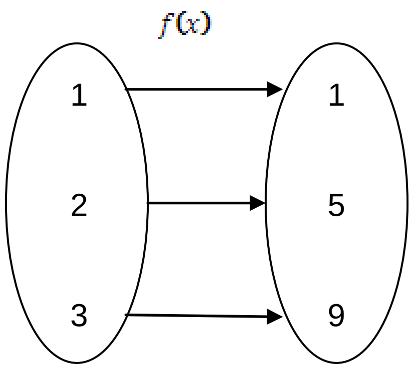
At the end of this lesson you will be able to:

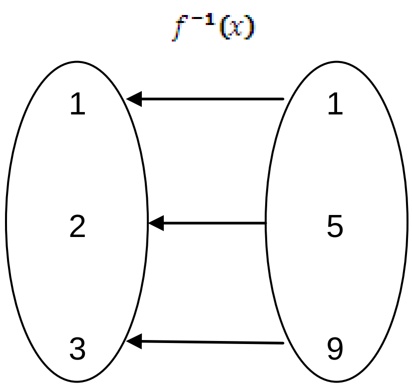
a) Determine the inverse of various functions
b) Interpret and make use of the functional notation $f^{-1}(x)$.

8.5 Inverse of a function

If a function $f(x)$ takes you from the domain elements to the range elements, then the inverse of the function or $f^{-1}(x)$, will take you from the range elements to the domain elements.

Example:





Determining the inverse of a function.

Given a function you can determine the inverse by interchanging x and y and then making y the subject.

Determine the inverse of the function $f(x) = 4x - 3$.

Solution:

First this can be written as $y = 4x - 3$.

By interchanging x and y we have:
$x = 4y - 3$

Now we have to make y the subject of the equation.

$x + 3 = 4y$

Then $y = \frac{x+3}{4}$

Hence $f^{-1}(x) = \frac{x+3}{4}$

From the mapping diagram above we saw that 2 was mapped on to 5 since

$f(2) = 4(2) - 3$

$= 8 - 3$

$= 5$.

The inverse will map 5 back to 2. Therefore

$f^{-1}(x) = \frac{5+3}{4}$

$= \frac{8}{4}$

= 2.

The inverse of composite functions can also be found.

Example: The functions f and g are defined by:

$f(x) = \frac{2x+3}{x-1}$ and $g(x) = 3x + 1$. Determine the inverse of $(fg)^{-1}(x)$

Solution:

$$fg(x) = \frac{6x + 5}{3x}$$

To determine $(fg)^{-1}(x)$ follow the same steps as before.

$$y = \frac{6x + 5}{3x}$$

Interchange x and y .

$$x = \frac{6y + 5}{3y}$$

Make y the subject of the equation.

$$3xy = 6y + 5$$

$$3xy - 6y = 5$$

$$3xy - 6y = 5$$

$$y = \frac{5}{3x - 6}$$

$(fg)^{-1}(x) = \frac{5}{3x - 6}$.

ACTIVITY 1

Given that $h(x) = \frac{5x + 1}{3x - 1}$, state an expression for $h^{-1}(x)$. (Ex 14b page 806)

The inverse of a function can be used to find elements in the domain.

Example: Solve the equation $\frac{3x + 5}{x + 5} = 2$.

First determine the inverse. Then substitute 2 into the inverse to get the solution.

Solution: $y = \frac{3x + 5}{x + 5}$

Interchange x and y .

$x = \frac{3y + 5}{y + 5}$

Make y the subject.

$x(y + 5) = 3y + 5$

$xy + 5x = 3y + 5$

$xy - 3y = 5 - 5x$

$y(x - 3) = 5 - 5x$

$y = \frac{5 - 5x}{x - 3}$

$$f^{-1}(x) = \frac{5-5x}{x-3}$$

Therefore

$$f^{-1}(2) = \frac{5-5(2)}{2-3}$$

$$= \frac{5-10}{2-3}$$

$$= \frac{-5}{-1}$$

$= 5$. Therefore $x = 5$.

ACTIVITY 2

Determine the value of x for which $\frac{x+3}{x-1} = 2$

ASSESSMENT

1. Two functions , g and h , are defined as

$$g: x \rightarrow \frac{2x+3}{x-4}$$ and

$$h: x \rightarrow \frac{1}{x}.$$

Calculate

(i) the value of $g(7)$

(ii) the value of x for which $g(x) = 6$.

Write expressions for

(iii) $hg(x)$

(iv) $(hg)^{-1}(x)$

(v) $g^{-1}(x)$.

Conclusion

In this lesson we looked at finding inverses of functions. We also observed that the inverse of composite functions can be found using the same steps. It is important also to note that elements in the domain can be found using the inverse of the function. In the lesson that follows we will look at variation.